

**Exercise 58**

Find an equation of the tangent to the curve at the given point.

$$y = \frac{x^2 - 1}{x^2 + 1}, \quad (0, -1)$$

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**Solution**

The aim is to find the slope of the tangent line at  $x = 0$ . Take the derivative of  $y$ .

$$\begin{aligned} y' &= \frac{d}{dx} \left( \frac{x^2 - 1}{x^2 + 1} \right) \\ &= \frac{\left[ \frac{d}{dx}(x^2 - 1) \right] (x^2 + 1) - \left[ \frac{d}{dx}(x^2 + 1) \right] (x^2 - 1)}{(x^2 + 1)^2} \\ &= \frac{(2x)(x^2 + 1) - (2x)(x^2 - 1)}{(x^2 + 1)^2} \\ &= \frac{4x}{(x^2 + 1)^2} \end{aligned}$$

Plug in  $x = 0$ .

$$y'(0) = \frac{4(0)}{(0^2 + 1)^2} = 0$$

Use the point-slope formula with this slope and the given point  $(0, -1)$  to get the equation of the tangent line.

$$y - (-1) = 0(x - 0)$$

$$y + 1 = 0$$

$$y = -1$$

Below is a graph of the curve and its tangent line at  $(0, -1)$ .

